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TOWARDS A THEORY OF CHOKE PISTONS,(U)

JAN 79 L A VAYNSHTEYN

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by

L.A. Vaynshteyn

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

Towards a Theory of Choke Pistons *

L. A. Vaynshteyn

The choke (reactive) pistons in the coaxial line, depicted in Fig. 1, were investigated. These pistons ensure almost complete reflection in a certain frequency range, whereas direct contact between the exterior and interior conductor of the line is absent. Both the elementary theory of these pistons, based on telegraph equations, and the more accurate electrodynamic theory, in which diffraction on the opened end of the piston is taken into account (Fig. 1, b and c), are discussed. A physical analysis of the blocking effect of the choke pistons, and also an analysis of the corrections, which appear in the transition from elementary to electrodynamic theory, are presented.

Sec. 1. Elementary Theory of Choke Pistons

Choke pistons of various types are described and their elementary theory is discussed in the literature (see for example [1]). The choke piston, represented in Fig. 1, a, by analogy with [1] can be called a capacitive piston, the one shown in Fig. 1, b, can be called a cap piston, and that in Fig. 1, c, can be called a bucket plunger. It is necessary to keep in mind here, that in contrast to the "pure" choke pistons, described in [1], the pistons, shown in Fig. 1, have contact with the interior conductor of the coaxial line. Moreover, it is evident, that the operation of the pistons, represented in Fig. 1, b and c, is completely equivalent: in accordance with the reciprocity theorem the trans-

* This work was accomplished in 1957.

mission coefficient of a wave, incident on a piston along a coaxial line from the right and from the left in Fig. 1, b or c, should be identical. Thus, the operating principle and the theory of a cap piston and a bucket piston are identical, and one should not be surprised by the identity of the numerical results in the calculation of both pistons (see [1], page 361).

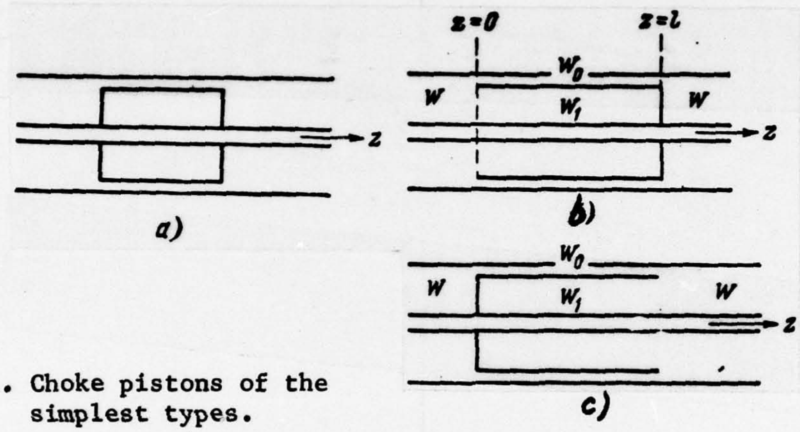

Fig. 1. Choke pistons of the simplest types.

Let us calculate the transmission of a wave, "incident" on a cap (Fig. 1, b) from the left. Let us designate via W the wave impedance of the coaxial line, via W_0 - the wave impedance of the section of the coaxial line, formed by the lateral surface of the piston and the exterior conductor of the coaxial line, via W_1 - the wave impedance of the line, formed by the lateral surface of the piston and the interior conductor. In accordance with the theory of long lines, when $z < 0$, current J and voltage U to the left of the cap in the line are equal to

$$J = e^{ikz} + Re^{-ikz}, \qquad U = W(e^{ikz} - Re^{-ikz}); \tag{1.01}$$

where for simplicity's sake the amplitude of the current of the incident wave is taken as unity; via R is designated the reflection factor, the time dependence $e^{-i\omega t}$, $k = \omega/c = 2\pi/\lambda$.

In the section $0 < z < l$, where the piston is located, it is necessary to distinguish current J_0 and voltage U_0 in the gap with wave impedance W_0:

$$J_0 = Ae^{ikz} + Be^{-ikz}, \qquad U_0 = W_0(Ae^{ikz} - Be^{-ikz}) \tag{1.02}$$

and current J_1 and voltage U_1 in ther interior line with wave impedance W_1:

$$J_1 = C[e^{ik(z-l)} + e^{-ik(z-l)}], \qquad U_1 = W_1C[e^{ik(z-l)} - e^{-ik(z-l)}]. \tag{1.03}$$

Here A, B and C are constants; the expressions of (1.03) are written taking into account boundary condition $U_1 = 0$ on the "bottom" of the cap, when $z = l$. When $z > l$ we have

$$J = Te^{ikz}, \qquad U = WTe^{ikz}, \tag{1.04}$$

where T is the transmission coefficient.

The values of A, B, C, R and T are determined from the five boundary conditions

$$\left.\begin{aligned} J = J_0 = J_1, \quad & U = U_0 + U_1 \quad \text{when } z = 0, \\ J_0 = J_1, \quad & U_0 = U \quad \text{when } z = l, \end{aligned}\right\} \tag{1.05}$$

the first of which expresses the current continuity on the interior and the exterior conductor of the coaxial line, and also the absence of charge accumulation on the edge of the piston. The solution of these equations gaves the following expression for the transmission coefficient:

$$T = \frac{2e^{-i\vartheta}}{\cos\vartheta\left[2 - m_1 \operatorname{tg}^2\vartheta - i\left(m + \frac{m_1}{m} + \frac{1}{m}\right)\operatorname{tg}\vartheta\right]}, \tag{1.06}$$

where

$$\vartheta = kl, \quad m = \frac{W}{W_0}, \quad m_1 = \frac{W_1}{W_0}. \tag{1.07}$$

Expression (1.06) shows, that

$$T = 0 \text{ when } \vartheta = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots \tag{1.08}$$

and with any m and m_1; however, with greater values of m and m_1 coefficient T takes rather small values in a larger range of values of ϑ. The curve, which gives the dependence of $|T|^2$ (in decibels) on ϑ, is given in work [1] on page 359 when $m_1 \approx m = 18{,}6$.

If a wave is incident on a cap from the right (Fig. 1,c) or on a bucket (Fig. 1, c) from the left, then analogous calculation leads to the same formula (1.06). Thus, these pistons at the optimum frequency (when $\vartheta = \pi/2$) give complete reflection, in contrast, for example, to a capacitive piston (Fig. 1, a), which also when $\vartheta = \pi/2$ gives a certain transmission - the lesser, the small the gap is. The physical reason for this difference will be discussed in Sec. 2.

Let us note, that formula (1.06) and all the conclusions following from it remain valid, if the entire section $0 < z < l$ is filled with a dielectric; in this case it is necessary to consider $\vartheta = k\sqrt{\varepsilon}\, l$ and to take into account the effect of the dielectric on wave impedances W_0 and W_1. It is optional to consider the lateral wall of the piston infinitely thin: it can have finite thickness. If we designate the interior and exterior radii of the coaxial line by a and b and the interior and exterior radii of the lateral wall of the piston by r_a and r_b, then in absolute units

$$W_0 = \frac{2}{c\sqrt{\varepsilon}} \ln\frac{b}{r_b}, \quad W_1 = \frac{2}{c\sqrt{\varepsilon}} \ln\frac{r_a}{a} \tag{1.09}$$

and for the transition to practical units it is necessary to replace 2/c by 60 ohms. When $r_a = r_b = r_0$ and $\varepsilon = 1$ these formulas take the following form

$$W_0 = \frac{2}{c}\ln\frac{b}{r_0}, \qquad W_1 = \frac{2}{c}\ln\frac{r_0}{a}, \qquad W = W_0 + W_1 = \frac{2}{c}\ln\frac{b}{a}. \tag{1.10}$$

Subsequently, we will have this simplest case in mind, although filling with a dielectric can be favorable from the point of view of decreasing the length of the piston l and increasing m and m_1.

Sec. 2. The Physical Meaning of the Formulas for Choke Pistons

For explaining the physical meaning of the obtained formulas let us examine the auxiliary problem of the branching of a coaxial line with wave impedance W into two lines with impedances W_0 and W_1, respectively (Fig. 2). Let a wave along line W_0 come to the plane of branching z = 0; then when $z < 0$ we have

$$\left.\begin{aligned} J_0 &= e^{ihz} - Qe^{-ihz}, & U_0 &= W_0(e^{ihz} + Qe^{-ihz}), \\ J_1 &= Se^{-ihz}, & U_1 &= -W_1Se^{-ihz}, \end{aligned}\right\} \tag{2.01}$$

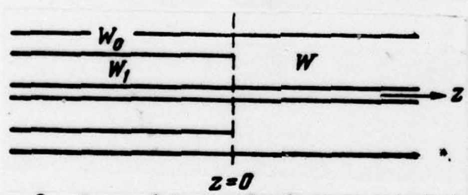

Fig. 2. Branching of the coaxial line

and when $z > 0$

$$J = Pe^{ihz}, \quad U = WPe^{ihz}, \tag{2.02}$$

where Q is the reflection factor, P is the coefficient of transmission, and value S can be called the coefficient of rotation, since the wave in line W_1 is excited by the rotation of the incident wave around the edge. The boundary conditions when z = 0 give

$$P = S = \frac{2}{m + m_1 + 1}, \quad Q = \frac{m + m_1 - 1}{m + m_1 + 1}. \tag{2.03}$$

If a wave is incident on line W_1, then when $z<0$

$$\left.\begin{array}{ll} J_1 = e^{ikz} - Q' e^{-ikz}, & U_1 = W_1(e^{ikz} + Q' e^{-ikz}), \\ J_0 = S' e^{-ikz}, & U_0 = -W_0 S' e^{-ikz}, \end{array}\right\} \tag{2.04}$$

and when $z>0$

$$J = P' e^{ikz}, \quad U = WP' e^{ikz}. \tag{2.05}$$

The new coefficients Q' (of reflection), P' (of transmission) and S' (of rotation) are equal to

$$P' = S' = \frac{2m_1}{m+m_1+1}, \quad Q' = \frac{m+1-m_1}{m+1+m_1}. \tag{2.06}$$

In both cases the transmission coefficient P (or P') is equal to the coefficient of rotation S (or (S'); these coefficients characterize the current in the lines (and not the voltage) and their equality ensues from the current continuity on the interior conductor (Fig. 2), since when $z>0$ this current is determined by the wave which has passed, and when $z<0$ by the wave which has turned.

With the aid of equality P = S it is possible to explain the operation of the piston in Fig. 1, b, in the following manner, and, in particular, equality T = 0 when $\vartheta = \pi/2$. A wave running along gap W_0 , when $z = l$ gives the origin to the passed and the turned waves; the turned wave is reflected from the "bottom" z = 0 (without a reversal in current phase) and goes to plane $z = l$ with additional phase 2ϑ and when $\vartheta = \pi/2$ completely suppresses the passed wave, since their amplitudes are equal.

If a wave is incident on the same piston from the right, or on the cap piston (Fig. 1, b) from the left, then it passes first of all (without distur-

bance, if the wall is infinitely thin and there is no dielectric) into lines W_0 and W_1, forming there travelling waves with equal current amplitudes. A wave in line W_1, being reflected from the "bottom" $z = l$, gives origin to the reflected wave in the line and, turning into line W_0, compensates the wave, which just passed into this line; when $\vartheta = \pi/2$ there is complete compensation and absence of a field in line W_0. Actually, from the formulas of Section 1 we have in this case A = B = 0.

Thus, the principle of the blocking effect of the cap piston (Fig. 1, b and c) consists in the "branching" of the travelling wave and in such matching of the phase difference, that the passed wave became zero. The operating principleof the capacitive piston (Fig. 1, a) is simpler and, therefore, it does not give complete reflection.

Let us note, that when the formulas of (1.10) are correct the values of (2.03) and (2.06) are equal to

$$P = Q' = S = \frac{\ln \frac{b}{r_0}}{\ln \frac{b}{a}}, \quad P' = Q = S' = \frac{\ln \frac{r_0}{a}}{\ln \frac{b}{a}}. \qquad (2.07)$$

Sec. 3. Concerning Corrections in the End Effects

Elementary theory, discussed above and based on the theory of long lines, leads to the conclusion, that the reflection from the piston will be complete when $\vartheta = \pi/2$ and $l = \lambda/4$. Experience shows (see [1], page 363), that optimum operation of the piston occurs at somewhat lesser ϑ and l.

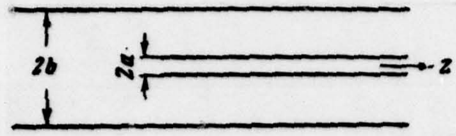

Fig. 3. **Passage of a coaxial line into a waveguide**

This discrepancy is connected with the fact, that we used the theory of long lines, which gives only approximate results. The character of the corrections, which strict theory should give, can be predicted, by examining the following examples.

In acoustics the natural oscillations of pipes (organ pipes) open at one end, which connect with surrounding space, and closed at the other end with a piston, are studied. Elementary theory leads to the conclusion, that resonance in such systems begins when $l = \lambda/4$ (l is the length of the pipe, λ is the wavelength), however, strict theory, which takes into account diffraction at the open end of a pipe, leads to a more precise condition of resonance

$$l = \frac{\lambda}{4} - \alpha, \tag{3.01}$$

where α — the so-called correction for the open end, for rather long waves (λ is considerably greater than radius a of the pipe), is equal to

$$\lambda = 0{,}613a \tag{3.02}$$

(see [2], Chap. III).

It is also possible to examine a coaxial line, passing into a round waveguide (Fig. 3). If the waves cannot be propagated in the round waveguide in the given frequency range, then a wave, being propagated in a coaxial line in the direction of the "open end" z = 0, should be completely reflected. Thus, when $z > 0$, by disregarding the damped waves (those having noticeable ampli-

tudes only when $|z| \sim b$, where b is the radius of the exterior conductor), we will have

$$J = e^{-ikz} + Re^{ikz}, \quad U = W(e^{-ikz} + Re^{ikz}), \tag{3.03}$$

where R, the current reflection factor, is equal in absolute value to unity and thus can be represented in the following form

$$R = -e^{2ik\alpha}. \tag{3.04}$$

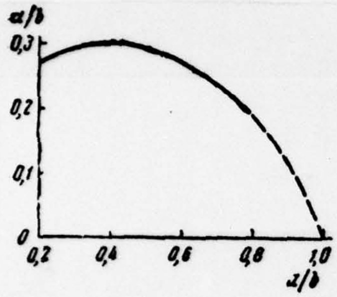

Fig. 4. The dependence of α on the ratio a/b for a coaxial resonator.

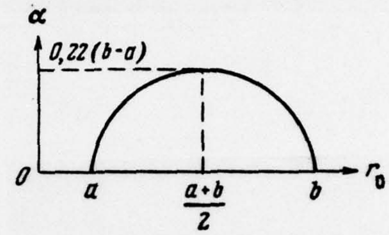

Fig. 5. The dependence of α on r_0 for a choke piston.

Parameter α for rather long waves $(\lambda \gg b)$ is determined only by the geometric parameters a and b (Fig. 3); it is depicted in Fig. 4, taken from [3], and it is possible to find the derivation of the standard working formulas in [4].

The formulas of (3.03) and (3.04) show, that, when $z = l$ (Fig. 3) by placing a metal partition, we obtain a coaxial resonator, the resonance condition in which has the form (3.01). In comparing this formula with the experiment it is necessary to keep in mind, that in theory, the interior conductor was taken in the form of a hollow tube, and not in the form of a solid cylindrical body, and this in the expression for α can give a correction of the order of a; when $a \ll b$ it is possible to disregard this correction.

We will show in the following section, that the strict theory of a cap piston also leads to the relationship (3.01), which is the condition of complete

reflection. The dependence of the correction on end effect α on geometry is depicted (roughtly) in Fig. 5; it is possible to calculate this value more accurately with formula (4.12)

Sec. 4. Strict calculation

Let us examine the electromagnetic waves in the system, depicted in Fig. 6; a is the radius of the interior conductor, b is the radius of the exterior conductor, r_0 is the radius of the intermediate conductor, which occupies section $0 < z < \infty$. We will consider all conductors ideally conducting and limited by waves, the currents of which flow only in the longitudinal (z-th) direction and the fields of which possess symmetry of rotation and are expressed by a single component A_z of the vector potential, which satisfies the wave equation

$$\frac{\partial^2 A_z}{\partial z^2} + \frac{\partial^2 A_z}{\partial r^2} + \frac{1}{r}\frac{\partial A_z}{\partial r} + k^2 A_z = 0, \tag{4.01}$$

so that

$$E_z = -\frac{1}{ik}\left(\frac{\partial^2 A_z}{\partial z^2} + k^2 A_z\right), \quad E_r = -\frac{1}{ik}\frac{\partial^2 A_z}{\partial z \partial r}, \quad H_\varphi = -\frac{\partial A_z}{\partial r}. \tag{4.02}$$

Function A_z should be continuous when $r = r_0$ and become zero when $r = a$ and $r = b$, thus it is possible to represent it in the following form (see [4])

$$A_z = -\frac{\pi}{c}\int_C \frac{e^{iwz}F(w)\,dw}{J_0(vb)N_0(va) - N_0(vb)J_0(va)} \times$$
$$\times \begin{Bmatrix} [J_0(vr)N_0(va) - N_0(vr)J_0(va)][J_0(vb)N_0(vr_0) - N_0(vb)J_0(vr_0)] \\ [J_0(vr_0)N_0(va) - N_0(vr_0)J_0(va)][J_0(vb)N_0(vr) - N_0(vb)J_0(vr)] \end{Bmatrix}, \tag{4.03}$$

where in the the braces it is necessary to take the upper row when $r < r_0$, and the lower row, when $r > r_0$. By calculating current J, flowing along the intermediate conductor of radius r_0, we will obtain

$$J = \frac{cr_0}{2}\left(H_\varphi\Big|_{r=r_0+0} - H_\varphi\Big|_{r=r_0-0}\right) = \int_C e^{iwz}F(w)\,dw. \tag{4.04}$$

Fig. 6. Various cases of the branching of a coaxial line

Here C denotes the contour in the plane of complex variable w, which we will subsequently draw, so that it includes point w = − k below and then continues along the real axis (in the derivation we consider $\operatorname{Im} k > 0$, and we change to the real k only in the final formulas). The root of $\sqrt{k^2 - w^2}$, is designated by v, and we assume $\operatorname{Im} v > 0$ when $\operatorname{Im} k > 0$.

Function F(w) is found from the condition of the absence of current on the geometric continuation of the intermediate conductor

$$\int_C e^{iwz} F(w)\,dw = 0 \quad \overset{\text{when}}{\text{при}} \quad z < 0 \tag{4.05}$$

and from condition $E_z = 0$ on this conductor, which gives

$$\int_C e^{iwz} v \Psi(w) F(w)\,dw = 0 \quad \overset{\text{when}}{\text{при}} \quad z > 0, \tag{4.06}$$

where

$$\Psi(w) = i\pi v r_0 \times$$
$$\times \frac{[J_0(vr_0)N_0(va) - N_0(vr_0)J_0(va)][J_0(vb)N_0(vr_0) - N_0(vb)J_0(vr_0)]}{J_0(vb)N_0(va) - N_0(vb)J_0(va)} = \tag{4.07}$$
$$= \frac{\Psi_a(w)\Psi_b(w)}{\Psi_c(w)}$$

$$\left.\begin{aligned}\Psi_a(w) &= i\pi v\sqrt{r_0 a}\, e^{iv(r_0-a)}[J_0(vr_0)N_0(va) - N_0(vr_0)J_0(va)],\\ \Psi_b(w) &= i\pi v\sqrt{br_0}\, e^{iv(b-r_0)}[J_0(vb)N_0(vr_0) - N_0(vb)J_0(vr_0)],\\ \Psi_c(w) &= i\pi v\sqrt{ba}\, e^{iv(b-a)}[J_0(vb)N_0(va) - N_0(vb)J_0(va)].\end{aligned}\right\} \tag{4.08}$$

Functions Ψ, Ψ_a, Ψ_b and Ψ_c when $\operatorname{Im} v \to \infty$ tend towards unity, as is evident, for example, from the asymptotic expression for $\Psi(w)$:

$$\Psi(w) = -2i\frac{\sin v(r_0 - a)\sin v(b - r_0)}{\sin v(b-a)}. \tag{4.09}$$

If we designate by w_m the roots of equation $\Psi_a(w) = 0$, arranged in ascending order of the corresponding positive numbers v^a_m $(m = 1, 2, \ldots)$, if we introduce the analogous designations: w^b_m and v^b_m, w^c_m and v^c_m , then the numbers $\pm w^a_m, \pm w^b_m, \pm w^c_m$ are wave numbers of symmetrical electrical waves E_{0m} in the coaxial lines a, b, and c (Fig. 6, a). Subsequently, we will assume $\operatorname{Im} w^{a,b,c}_m > 0$, so that the wave numbers $w^{a,b,c}_m$ correspond to waves, damped in the positive direction of axis z. From formula (4.09) it is easy to obtain the approximate expressions

$$v^a_m = \frac{m\pi}{r_0 - a}, \quad v^b_m = \frac{m\pi}{b - r_0}, \quad v^c_m = \frac{m\pi}{b-a}, \tag{4.10}$$

which, strictly speaking, are asymptotic and are fulfilled when $m \to \infty$, but actually give good results even when m = 1 (compare [3], page 81).

The solution of the functional equations (4.05) and (4.06) can be found with the aid of functions $\Psi_1(w)$ and $\Psi_2(w) = \Psi_1(-w)$, which are holomorphic and do not become zero respectively in the upper $(\operatorname{Im} w \geqslant 0)$ and lower $(\operatorname{Im} w \leqslant 0)$ half-planes, the product of which $\Psi_1(w)\Psi_2(w)$ is equal to the given function $\Psi(w)$. Repeating the operations, analogous to those presented in article [4], we obtain

$$\left.\begin{aligned}\Psi_1(w) &= K\sqrt{k+w}\,e^{iwa_0}\prod_{m=1}^{\infty}\frac{\left(1+\frac{w}{w_m^a}\right)\left(1+\frac{w}{w_m^b}\right)}{1+\frac{w}{w_m^c}}\\ \Psi_2(w) &= K\sqrt{k-w}\,e^{-iwa_0}\prod_{m=1}^{\infty}\frac{\left(1-\frac{w}{w_m^a}\right)\left(1-\frac{w}{w_m^b}\right)}{1-\frac{w}{w_m^c}}\\ K &= \sqrt{\frac{\Psi(0)}{k}},\end{aligned}\right\} \tag{4.11}$$

where

$$a_0 = \frac{1}{\pi}[(b-a)\ln(b-a)-(r_0-a)\ln(r_0-a)- \\ -(b-r_0)\ln(b-r_0). \tag{4.12}$$

If we take function F(w) in the following form

$$F(w) = \frac{D}{(k+w)\sqrt{k-w}\Psi_2(w)} = \frac{1}{v\Psi(w)}\frac{D\Psi_1(w)}{\sqrt{k+w}} \tag{4.13}$$

and contour C, as was indicated above, then equations (4.05) and (4.06) are satisfied. In order to elucidate the physical meaning of the solution obtained, let us calculate currents J_a and J_b on the interior and exterior conductors of the coaxial line

$$\left.\begin{aligned}J_a &= \frac{ca}{2}H_\varphi\Big|_{r=a} = -\int_C e^{iwz}F(w)\frac{J_0(vr_0)N_0(va)-N_0(vr_0)J_0(va)}{J_0(vb)N_0(va)-N_0(vb)J_0(va)}dw,\\ J_b &= -\frac{cb}{2}H_\varphi\Big|_{r=b} = -\int_C e^{iwz}F(w)\frac{J_0(vb)N_0(vr_0)-N_0(vb)J_0(vr_0)}{J_0(vb)N_0(va)-N_0(vb)J_0(va)}dw.\end{aligned}\right\} \tag{4.14}$$

It is easy to transform these integrals into a series of residues; when $z > 0$ it is necessary to deform contour C on top and to consider poles $w = \pm k$ and $w_m^{a,b}$ of the integrand; we obtain

$$J_b = A(e^{-ikz} - Me^{ikz}), \quad J_a = A'(e^{-ikz} - Me^{ikz}), \tag{4.15}$$

where the constants A and A' are connected by the following relationship

$$\frac{A'}{A}=\frac{\ln\frac{b}{r_0}}{\ln\frac{r_0}{a}} \tag{4.16}$$

and coefficient M is equal to

$$M=e^{2ik\alpha_0}\prod_{m=1}^{\infty}\frac{1+\frac{k}{w_m^a}}{1-\frac{k}{w_m^a}}\cdot\frac{1+\frac{k}{w_m^b}}{1-\frac{k}{w_m^b}}\cdot\frac{1-\frac{k}{w_m^c}}{1+\frac{k}{w_m^c}} \tag{4.17}$$

In formulas (4.15)-(4.17) we can already consider wave number k real; if the frequency is sufficiently low, so that waveguide waves E_{0m} cannot be propagated in the coaxial line (and only the transverse wave with wave number k is propagated), then the wave numbers $w_m^{a,b,c}$ are purely imaginary $(w_m = i\,|w_m|)$ and it is possible to represent value (4.17) in the following form

$$M=e^{2ik\alpha}, \tag{4.18}$$

since the infinite product is equal in absolute value to identity. Employing the approximate formulas of (4.10), it is easy to represent the value of α, which determines M, in the following form

$$\alpha=\alpha_0+\frac{1}{k}\sum_{m=1}^{\infty}\left[\arcsin\frac{k(b-a)}{m\pi}-\arcsin\frac{k(r_0-a)}{m\pi}-\right.$$
$$\left.-\arcsin\frac{k(b-r_0)}{m\pi}\right]. \tag{4.19}$$

In the formulas of (4.15) the terms are not written out, i.e., the terms which correspond to the residues at points $w_m^{a,b}$, since they yield damped waves, which, when $z\gtrsim b$, it is possible to disregard. In calculating the integrals of (4.14) when $z<0$ with respect to the residues at poles $-w_m^c$, which lie below contour C, we will obtain a number of damped waves, which , when $z\lesssim -b$ it is possible to disregard. Disregarding the damped waves, as in the formulas

of (4.15). we can write

$$J_a = J_b = 0 \quad \overset{when}{\text{при}}\ z < 0. \tag{4.20}$$

The coaxial line is usually employed under the condition

$$kb \ll 1, \tag{4.21}$$

and then the data higher than the estimation of the region, occupied by damped waves, is applicable; in this case it is also possible to disregard the series in formula (4.19) and to assume $\alpha = \alpha_0$.

If we consider, that the voltages U_a and U_b in lines a and b, in accordance with the formulas of (4.15), can be represented in the following form

$$U_a = U_b = A_0(e^{-ikz} + Me^{ikz}), \tag{4.22}$$

where

$$A_0 = -W_aA' = -W_bA, \quad W_a = \frac{2}{c}\ln\frac{r_0}{a}, \quad W_b = \frac{2}{c}\ln\frac{b}{r_0}, \tag{4.23}$$

then the strict solution of the electrodynamic problem, obtained above, is interpreted physically by one of the following four methods.

A. In the system, depicted in Fig. 6, a, waves with current amplitudes A and A' advance to the right along lines a and b. Each of these waves is partially reflected from the plane of junction z = 0, partially turns into the other line. However, the wave is not excited in line c, more accurately, the waves, excited by each line a and b separately, completely compensate each other.

B. In line a, it is possible, when $z = l$, to place an ideally conducting partition (Fig. 6, b). The formulas of (4.17) show, that if l is selected in

accordance with formula (3.01), then this partition does not lead to excitation of the field (since it is placed where $U_a = 0$). Thus, a wave, arriving at the plane of junction $z = 0$ along line b, will be completely reflected from it (due to the effect of "cap" a) and does not pass farther, into line c.

C. Similarly, by placing a partition in line b (Fig. 6, c), we attain complete reflection of the wave, travelling along line a. The length of the corresponding "pocket" b should again be selected in accordance with formula (3.01).

D. Finally, by placing a solid partition $z = l$ (Fig. 6, d), we obtain a unique cavity resonator with a high figure of merit, the natural frequency of which is determined with formula (3.01). This system behaves like a resonator only when it is disturbed at points a or b. With simultaneous excitation at points a and b the system resonates only by the <u>difference</u> of the excitations, and the total excitation is freely propagated along line c. Upon excitation of a wave in line c it is reflected from partition $z = l$ without any resonance. A resonator with such properties can, probably, find application.

In conclusion, let us note, that the strict solution obtained makes it possible to more accurately calculate the coefficients of reflection, transmission and rotation, calculated with the aid of the elementary theory in Section 2. And notably, coefficients P and P' are equal to

$$P = \frac{\ln\frac{b}{r_0}}{\ln\frac{b}{a}}, \quad P' = \frac{\ln\frac{r_0}{a}}{\ln\frac{b}{a}}, \qquad (4.24)$$

i.e., precisely those, as in accordance with elementary theory [formula (2.07)],

and the other coefficients acquire an additional phase factor

$$Q' = S = Pe^{2ih\alpha}, \quad Q = S' = P'e^{2ih\alpha}, \tag{4.25}$$

which leads to the correction for the end effect α in the formula (3.01). The value of α, as was already noted, under condition (4.21) is practically equal to value α_0, determined by formula (4.12).

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